Engineering Geometry
Objectives

- Describe the importance of engineering geometry in design process.
- Describe coordinate geometry and coordinate systems and apply them to CAD.
- Review the right-hand rule.
- List major categories of geometric entities.
Objectives

- Explain geometric conditions that occur between lines.
- Explain tangent conditions between lines and curves.
- List and describe surface geometric form.
- Describe engineering applications of geometry.
Engineering Geometry

Engineering geometry is the basic geometric elements and forms used in engineering design. Engineering and technical graphics are concerned with the descriptions of shape, size, and operation of engineered products.
Engineering Geometry

Shape Description
- Shape description of an object relates the positions of its component geometric elements (e.g., vertices, edges, faces) in space.

Coordinate Space
- In order to locate points, lines, planes, or other geometric forms, their positions must first be referenced to some known position, called a reference point or origin of measurement.
Coordinate Space

The Cartesian coordinate system, commonly used in mathematics and graphics, locates the positions of geometric forms in 2-D and 3-D space.

- A 2-D coordinate system establishes an origin at the intersection of two mutually perpendicular axes, labeled X (horizontal) and Y (vertical).
Coordinate Space

- In a 3-D coordinate system, the origin is established at the point where three mutually perpendicular axes (X, Y, and Z) meet. The origin is assigned the coordinate values of 0,0,0.
Coordinate Space

- The right-hand rule is used to determine the positive direction of the axes. The right-hand rule defines the X, Y, and Z axes, as well as the positive and negative directions of rotation on each axes.
Coordinate Space

Polar coordinates are used to locate points in the X-Y plane. Polar coordinates specify a distance and an angle from the origin (0,0).
Cylindrical coordinates locate a point on the surface of a cylinder by specifying a distance and an angle in the X-Y plane, and the distance in the Z direction.
Spherical coordinates locate a point on the surface of a sphere by specifying an angle in one plane, an angle in another plane, and one height.
Coordinate Space

- **Absolute coordinates** are always referenced to the origin (0,0,0).
- **Relative coordinates** are always referenced to a previously defined location and are sometimes referred to as delta coordinates, meaning changed coordinates.
Coordinate Space

- The **world coordinate system** uses a set of three numbers \((x,y,z)\) located on three mutually perpendicular axes and measured from the origin \((0,0,0)\).

- The **local coordinate system** is a moving system that can be positioned anywhere in 3-D space by the user, to assist in the construction of geometry.
Coordinate Space
A **point** is a theoretical location that has neither width, height, nor depth. Points describe an exact location in space. Normally, a point is represented in technical drawings as a small cross made of dashes that are approximately 1/8" long. In computer graphics, it is common to use the word **node** to mean a point. A **locus** represents all possible positions of a point.
**Points**

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<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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<tbody>
<tr>
<td>Approximately 1/8° long</td>
<td>Extruded to form a line</td>
<td>Point node at the tangency of 2 curves</td>
<td>Point at the center of a circle</td>
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<tr>
<th>E1</th>
<th>F</th>
<th>E2</th>
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<tbody>
<tr>
<td>Point nodes at the end of a line</td>
<td>Point at the intersection of 2 lines</td>
<td>Locus of the centers of the circles</td>
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<th>E</th>
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<tr>
<td>Locus of centers</td>
<td>Locus of the centers of the circles</td>
<td>Locus of centers</td>
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A **line** is a geometric primitive that has length and direction, but not thickness. A line may be straight, curved, or a combination of these.
Lines Relations
A **curved line** is the path generated by a point moving in a constantly changing direction, or is the line of intersection between a 3-D curved surface and a plane.

- single-curved (circle, ellipse, parabola)
- double-curved (cylindrical helix, conical helix)
Tangent Conditions

A **tangent condition** exists when a straight line is in contact with a curve at a single point.
Tangent Conditions

In 3-D geometry, a tangent condition exists when a plane touches but does not intersect another surface at one or more consecutive points.
Tangent Conditions
Tangent Conditions
Circles

- A **circle** is a single-curved-surface primitive, all points of which are equidistant from one point, the center.
- A circle is also created when a plane passes through a right circular cone or cylinder and is perpendicular to the axis of the cone.
- The elements of a circle: diameter, radius, chord, circumference, secant, arc, tangent, concentric.
Circle
A **parabola** is the curve created when a plane intersects a right circular cone parallel to the side of the cone. A parabola is a single-curved-surface primitive. Mathematically, a parabola is defined as the set of points in a plane that are equidistant from a given fixed point, called a focus, and a fixed line, called a directrix.
Conic Curves

A hyperbola is the curve of intersection created when a plane intersects a right circular cone that makes a smaller angle with the axis than do the elements.
Conic Curves

An **ellipse** is a single-curved-surface primitive and is created when a plane passes through a right circular cone oblique to the axis, at an angle to the axis greater than the angle between the axis and the sides.
Conic Curves

- A **spiral** is a single-curved surface that begins at a point called a pole and becomes larger as it travels around the origin in a plane.
Conic Curves

A **cycloid** is the curve generated by the motion of a point on the circumference of a circle as the circle is rolled along a straight line in a plane.
Conic Curves

An **involute** is a spiral path of a point on a string unwinding from a line, circle, or polygon.
Doubled-curved lines

A double-curved line is a curve generated by a point uniformly moving at both an angular and a linear rate around a cylinder or cone.

- **cylindrical helix**
  - Spiral staircases, worm gear, drill bits, spring

- **conical helix**
Freeform Curves

- If the curves are created by smoothly connecting the control points, the process is called **interpolation**.
- If the curves are created by drawing a smooth curve that goes through most, but not all the control points, the process is called **approximation**.
A **spline curve** is a smooth, freeform curve that connects a series of control points. Changing any single control point will result in a change in the curve, so that the curve can pass through the new point.
Freeform Curves

- The **Bezier curve**, which uses a set of control points that only approximate the curve.
- The **B-spline curve**, which approximates a curve to a set of control points and does provide for local control.
Angles

- **Angles** are formed by the apex of two intersecting lines or planes
  - Straight
  - Right
  - Acute
  - Obtuse
  - Complementary
Planes

A *plane* is an infinite, unbounded, two-dimensional surface that wholly contains every straight line joining any two points lying on the surface.
Surfaces

A **surface** is a finite portion of a plane, or the outer face of an object bounded by an identifiable perimeter.
2-D Surfaces

- **Quadrilaterals** are four-sided plane figures of any shape. The sum of the angles inside a quadrilateral will always equal 360 degrees.
2-D Surfaces

A **polygon** is a multisided plane of any number of sides.
2-D Surfaces

- A **triangle** is a polygon with three sides. The sum of the interior angles equals 180 degrees.
Ruled Surfaces

**Single-curved surfaces** are generated by moving a straight line along a curved path such that any two consecutive positions of the generatrix are:

- either parallel (cylinder),
- intersecting (cone),
- tangent to a double-curved line (convolute).
Ruled Surfaces

A **cone** is a single-curved-surface primitive formed by a line (generatrix) moving along a curved path such that the line always passes through a fixed point, called the vertex.
Ruled Surfaces

A **cylinder** is a single-curved ruled surface formed by a vertical, finite, straight-line element (generatrix) revolved parallel to a vertical or oblique axis directrix and tangent to a horizontal circular or elliptical directrix. The line that connects the center of the base and the top of a cylinder is called the axis.
Ruled Surfaces

A **convolute** is a single-curved surface generated by a straight line moving such that it is always tangent to a double-curved line.
Ruled Surfaces

**polyhedron** is a symmetrical or asymmetrical 3-D geometric surface or solid object with multiple polygonal sides. The sides are plane ruled surfaces, and are called faces, and the lines of intersection of the faces are called the edges.
polygonal prism is a polyhedron that has two equal parallel faces, called its bases, and lateral faces that are parallelograms. The parallel bases may be of any shape and are connected by parallelogram sides. A line connecting the centers of the two bases is called the axis.
Ruled Surfaces

**pyramid** is a polyhedron that has a polygon for a base and lateral faces that have a common intersection point called a vertex. The axis of a pyramid is the straight line connecting the center of the base to the vertex.
warped surface is a double-curved ruled 3-D surface generated by a straight line moving such that any two consecutive positions of the line are skewed (not in the same plane).